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Strategic optimal portfolio choice with financial frictions

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Abstract

This paper provides a solution to the multiperiod asset allocation problem of risk averse individuals facing financial frictions. We develop theory and empirical methods to obtain estimates of the individual's optimal financial leverage and portfolio asset allocation. These decision variables are assumed to be parametric functions of macroeconomic and financial variables with such specifications tested using statistical methods. The empirical application to a tactical portfolio reveals three main findings: a) financial frictions limit the reaction of investors to changes in the investment opportunity set; b) individuals hold countercyclical leverage positions with the potential to reduce the volatility of debt over time; c) optimal portfolio weights and financial leverage are negatively related to the degree of investor's risk aversion and positively related to the investment horizon.

Key words: borrowing constraints; financial frictions; financial leverage; gmm estimation and testing; strategic optimal asset allocation.

JEL Codes: E32, E52, E62, G01.

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1 Introduction

The effect of financial frictions is extensively studied in different frameworks within the realm of macroeconomics and finance. A simple interpretation of financial frictions in optimal portfolio allocation conforms with the view that investors face borrowing frictions that limit their portfolio investment decisions and risk-taking behavior. The absence of short-selling practices is an example of such frictions. More generally, financial frictions in optimal portfolio theory can be linked to limits to arbitrage as in Shleifer and Vishny (1997), the imposition of margin requirements to be satisfied by arbitrageurs as in Aiyagari and Gertler (1999) and Gromb and Vayanos (2002) or the existence of borrowing frictions in convergence trading strategies as in Xiong (2001). In a similar framework, Brunnermeier and Pedersen (2009) provide a model that links an asset's market liquidity (i.e., the ease with which it is traded) and traders' funding liquidity (i.e., the ease with which they can obtain funding). In this model the ability of traders to provide market liquidity depends on the availability of funding. This funding, mainly determined by traders' capital and margin requirements, depends on the assets' market liquidity.

The existence of financial intermediation between economic agents provides an alternative channel to explain the effect of financial frictions on asset pricing and optimal portfolio allocation. In this framework Aiyagari and Gertler (1999) show that limited participation by different groups of individuals arises from the combination of specialization in asset trading and frictions in the ability of traders to obtain funds. The trader partly finances the investment positions in risky securities by directly borrowing from the household in the form of short-term riskless debt. He and Krishnamurthy (2013) also take the view that the investor is a financial intermediary but explore, instead, the effects of frictions on raising equity capital rather than debt.

In most of the literature discussed above the study of financial frictions is performed over multiple periods providing in most cases a dynamic analysis of individuals' consumption and optimal portfolio decisions. Xiong (2001), for example, assumes convergence traders to maximize an additively separable logarithmic utility function within an infinite horizon model. Gromb and Vayanos (2002) maximize the investment positions of arbitrageurs and two types of constrained investors over a finite multiperiod model; and He and Krishnamurthy (2013) study the optimal consumption and portfolio decision of infinitely-lived specialists maximizing a separable constant relative risk aversion (CRRA) utility function. The solutions presented in the literature to these multiperiod models rely in most cases on numerical approximations and simulations. This is the case, for example, in Xiong (2001) that presents a numerical solution to discuss the amplification mechanisms caused by the wealth effect of convergence trades and simulation exercises to study the stationary distribution of the equilibrium. He and Krishnamurthy (2013) use calibration methods to replicate the asset market behavior during crises. Aiyagari and Gertler (1999) derive an analytical solution for a deterministic version of their model and a simple numerical computation of the stochastic version. A similar exercise is carried out in Brunnermeier and Sannikov (2014) to compute the equilibrium price and optimal allocation of capital over an infinite horizon. Multiperiod maximization problems have also been discussed in the optimal portfolio allocation literature studying strategic, long-term, investment behavior. This literature, initiated by Merton (1969, 1971, 1973), and developed in detail by Brennan et al (1997) and Campbell and Viceira (1999, 2001, 2002), highlights the difficulty in finding analytical solutions when the number of periods in the maximization problem is higher than one and invokes a dynamic solution.

Our aim in this paper is to provide a satisfactory solution to the strategic asset allocation problem of long-term risk-averse individuals. Individuals in our model are financially constrained. Their problem is dynamic in several dimensions: first, they face a consumption and investment decision over a multiperiod horizon; second, the conditional distribution of the universe of asset returns varies over time entailing a time varying investment opportunity set and the existence of hedging demands on risky assets; and third, their optimal choice of debt raised in credit markets can change over time depending on the macroeconomic environment and their own equity. Financial frictions are introduced in this framework by assuming that investors partly finance their investment portfolios by borrowing from credit markets. On the firm level, papers such as Townsend (1979), Bolton and Scharfstein (1990), and DeMarzo and Sannikov (2006) explain why violations of Modigliani-Miller assumptions lead to bounds on the individuals' borrowing capacity, as well as restrictions on risk sharing. The wealth distribution among agents matters for the allocation of productive resources and overall economic activity. On the investor's level, Modigliani and Miller (1958)'s irrelevance proposition implies that investors are indifferent between financing their investment portfolios with their own equity or borrowed funds. Market imperfections, existence of transactions costs, and borrowing constraints established by regulatory capital requirements are among the factors that can make investors' funding decisions dependent on capital structure.

In this setup we entertain two different types of frictions. First, we assume lenders to require borrowers to finance investment positions in risky securities using their own equity as well as the proceeds of borrowing debt. This condition implies a debt ratio strictly smaller than one and limits the individual's use of leverage to be some multiple of his equity. In this context the borrower can still adjust his optimal leverage to the economic environment. We exploit this property and propose a parametric specification of the dynamics of the leverage parameter driving the amount of debt raised by the individual in each period. In particular, we propose a logarithmic specification that accommodates procyclical as well as countercyclical leverage dynamics. The cyclical character of the individual's leverage position is dictated by a set of parameters that are optimally chosen by the individual from the maximization of his multiperiod objective function. The other type of friction that we entertain in this paper sets an upper bound on the amount of leverage allowed to the individual in each period. In this alternative setup individuals' debt varies according to the evolution of their equity and cannot adjust depending on the macroeconomic environment. This financial friction is more binding than in the dynamic case in the sense that the individual has no leeway to optimally choose the amount of leverage in each period. Furthermore, debt is forced to be procyclical, increasing in economic expansions and decreasing in recessionary periods.

We assess the implications of these two types of financial frictions on the dynamic optimal portfolio decision of long-term investors. In order to convert the complex multiperiod maximization exercise intrinsic to the individual's strategic asset allocation problem into an analytically tractable problem, we consider a parametric linear portfolio policy rule that relates the state variables that drive the time-varying investment opportunity set to the individual's dynamic optimal portfolio weights. This approach to solving the strategic asset allocation problem provides an overidentifed system of equations obtained from the first order conditions of the investor's optimization problem over multiple periods. The overidentification property entails a natural empirical representation of the system of equations that can be used to apply GMM methods for estimating the model parameters and testing the parametric specifications of the optimal portfolio weights and functional form of the dynamic leverage parameter. Similar estimation and testing strategies are proposed by Hansen (1982), Cochrane (1996) and Aït-Sahalia and Brandt (2001) in different frameworks.

Our empirical analysis for a portfolio of stocks, bonds and cash reveals that individuals that optimize their leverage position with respect to the economic environment increase leverage during recessions and decrease it during economic expansions. This countercyclical leverage strategy has the potential to reduce the volatility of the debt process over time if the return on individual's equity is positively correlated with the economic cycle. The empirical analysis also reveals a more aggressive reaction of the optimal leverage position to the economic environment as the degree of relative risk aversion falls and the investment horizon increases. The second scenario characterized by a fixed and exogenously determined financial leverage position also reveals important insights on the relationship between the optimal portfolio allocation, the individual's financial leverage position, relative risk aversion and the investment horizon. More specifically, the existence of this type of financial frictions entails a monotonically decreasing relationship between the magnitude of the parameters driving the portfolio weights and the amount of leverage allowed in the individual's financial position. This empirical observation suggests that the exposure of the portfolio weights to the state variables driving the dynamic asset allocation becomes less responsive to variations in market conditions as the financial position of the individual becomes more levered. The existence of borrowing constraints increases the traders' effective degree of risk aversion by increasing the share of wealth invested on the risk-free asset. Furthermore, how effectively risk averse traders act also depends on the individual's relative risk aversion coefficient and the investment horizon characterizing his multiperiod objective function.

Our framework is similar to the settings presented in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) for explaining asset prices during crisis periods. The partial equilibrium model that we solve in this paper can also be applied to other related frameworks modeling investors' optimal behavior over long periods such as Xiong (2001) and Gromb and Vayanos (2002). Our description of the financial frictions faced by strategic investors is also similar to the borrowing constraints discussed in Kiyotaki and Moore (1997), Aiyagari and Gertler (1999), and more recently, He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014).

The rest of the article is structured as follows. Section 2 introduces the model and solves the multiperiod maximization problem with respect to the optimal portfolio allocation and the individual's dynamic leverage position. Section 3 discusses suitable econometric methods to estimate the model parameters and presents a statistical test to assess the correct specification of the parametric portfolio policy rule and the functional form of the individual's leverage position. Section 4 presents an application to empirically assess the relationship between the strategic optimal portfolio allocation, the different leverage positions defining each type of financial friction, the coefficient of relative risk aversion and the individual's investment horizon for a portfolio of stocks, bonds and cash. Section 5 concludes.

2 The Model

The model studies the consumption and investment decision of an agent who maximizes the expected utility of consumption (c_t) over a potentially infinite horizon model. Assume that the multiperiod utility function is additively time separable and exhibits constant relative risk aversion. An investor maximizes

$$\sum_{j=0}^{K} \beta^j E_t \left[\frac{c_{t+j}^{1-\gamma}}{1-\gamma} \right], \tag{1}$$

with $\gamma > 0$, $\gamma \neq 1$, and K an arbitrarily large number denoting the number of periods considered by the investor in his optimal consumption plan. Two parameters describe CRRA preferences. The discount factor β measures patience, the willingness to give up consumption today for consumption tomorrow. The coefficient γ captures risk aversion, the reluctance to trade consumption for a fair gamble over consumption today.

In a framework characterized by the absence of financial constraints the investor's strategic optimal portfolio allocation is obtained from maximizing the objective function (1) subject to the following budget constraint:

$$w_{t+i-1}(1 + r_{p,t+i}(\alpha_{t+i-1})) = c_{t+i} + w_{t+i}, \tag{2}$$

where w_{t+i} denotes individual's wealth and c_{t+i} denotes consumption. Implicitly, this budget constraint implies that the individual consumes in each period a quantity smaller than the proceeds of the investment portfolio such that there is always positive equity on the individual's financial position that can be subsequently invested on the portfolio. The investment portfolio is defined by a vector of weights $\alpha_t = (\alpha_{f,t}, \alpha_{1,t}, \ldots, \alpha_{m,t})'$ with $\alpha_{f,t} + \sum_{h=1}^{m} \alpha_{h,t} = 1$ where $\alpha_{f,t}$ denotes the allocation of wealth to the risk-free asset and $\alpha_{h,t}$ with $h = 1, \ldots, m$ are the allocations to the m risky assets. Following He and Krishnamurthy (2013), and most of the related literature on asset allocation with financial frictions, we do not impose short selling restrictions on the optimal portfolio weights implying that $\alpha_{h,t}$ can be positive or negative reflecting long or short positions on the risky assets, respectively. Then

$$r_{p,t+i}(\alpha_{t+i-1}) = r_{f,t+i} + r^{p}_{e,t+i}(\alpha_{t+i-1}), \tag{3}$$

with $r^{p}_{e,t+i}(\alpha_{t+i-1}) = \alpha'_{t+i-1} r_{e,t+i}$, where $r_{e,t+i} = (r_{1,t+i} - r_{f,t+i}, \ldots, r_{m,t+i} - r_{f,t+i})'$ is the vector of excess returns on the m risky assets.

2.1 Parametric portfolio policy rule

In order to convert the individual's multiperiod maximization exercise into an analytically tractable problem we assume a parametric linear portfolio policy rule for describing the dynamic optimal portfolio asset allocation. In particular, we follow the parametric methodology introduced in the seminal contributions of Aït -Sahalia and Brandt (2001), Brandt and Santa-Clara (2006) and Brandt et al. (2009), and entertain the following specification

$$\alpha_{h,t+i} = \lambda'_h z_{t+i}, \;\; h = 1, \ldots, m, \tag{4}$$

with $z_t = (1, z_{1,t}, \ldots, z_{n,t})'$ a set of state variables describing the evolution of the economy and $\lambda_h = (\lambda_{h,1}, \ldots, \lambda_{h,n})'$ the corresponding vector of parameters. Time variation of the optimal asset allocation is introduced through the dynamics of the state variables. This specification of the portfolio weights has two main features. First, it allows us to study the marginal effects of the state variables on the optimal portfolio weights through the set of parameters λ, and second, it avoids the introduction of time consuming stochastic dynamic programming methods. As a byproduct, this specification of the optimal portfolio weights could accommodate arbitrarily long horizons in the individual's objective function. A potential downside of this parametric approach is to force the individual's optimal portfolio policy rule to be linear and with the same parameter values over the long term horizon. Nevertheless, for finite horizon ($K < \infty$) objective functions, more sophisticated models can be developed that entertain different parametric portfolio policy rules for different investment horizons $i = 1, \ldots, K$. This approach significantly increases the computational complexity of the methodology and is beyond the aim of this study that focuses on the analysis of the effect of financial frictions on optimal asset allocation.

2.2 Financial frictions

Financial frictions are introduced in this framework by entertaining the possibility of funding investment portfolios through borrowing from credit markets. In this setup, equation (2) is modified to entertain the following budget constraint:

$$\widetilde{w}_{t+i-1}(1 + r_{p,t+i}(\alpha_{t+i-1})) + b_{t+i} = c_{t+i} + \widetilde{w}_{t+i} + b_{t+i-1}(1 + r_{b,t+i}) \tag{5}$$

where $\widetilde{w}_{t+i}$ denotes financial wealth and $b_{t+i} \geq 0$ is the amount of debt borrowed in that period. The variable $\widetilde{w}_{t+i}$ is different from the equity held by the individual in their financial position that is now defined as $w_{t+i} = \widetilde{w}_{t+i} - b_{t+i}$. The debt raised by the individual is fully repaid in each period at an interest rate $r_{b,t+i}$. Following most of the literature on asset allocation with financial frictions, see Aiyagari and Gertler (1999) and He and Krishnamurthy (2013) among many others, the empirical application will consider a borrowing interest rate that is exogenously determined and given by the risk-free rate.

The key financial friction entertained in this paper is to assume that borrowers finance their investment positions in risky securities using their own equity as well as the proceeds of borrowing debt. This condition implies a debt ratio strictly smaller than one and limits the individual's use of leverage to be some multiple of his equity. More formally, let $\eta_t = b_t/w_t$ define the individual's financial leverage position, then

$$b_{t+i} = f(\eta_{t+i})\widetilde{w}_{t+i}, \text{ with } f(\eta_{t+i}) = \frac{\eta_{t+i}}{1+\eta_{t+i}}, \tag{6}$$

and $0 < f(\eta) < 1$, by construction. It is worth noting that the existence of a debt ratio strictly smaller than one does not restrict the amount of leverage in the individual's financial position. One of the distinctive contributions of this article is, hence, to entertain a parametric specification for the dynamics of this parameter over time. In contrast to Kiyotaki and Moore (1997) that consider an endogenous leverage position that changes depending on the individual's asset holdings, our characterization of the individual's dynamic leverage position is with respect to the economic environment. This assumption entails a rich representation of the debt process b_t in terms of the economic environment, through the leverage position, and the individual's equity. In this context an increase in the contribution of debt to finance the investment portfolio can be due to a change in the macroeconomic landscape or an improvement on individual's equity. To do this, we entertain the following parametric specification of the process η_t, determined by a vector of business cycle variables, x_t, and a vector of parameters, κ, that are optimized by the individual in order to establish the optimal choice of financial leverage in each period:

$$\eta_{t+i} = \ln\left(1 + \exp(\kappa' x_{t+i})\right). \tag{7}$$

This function implies a positive relationship between the leverage parameter and the business cycle variable x_t if κ is positive and a negative relationship if κ is negative. As an illustrative example, we consider hereafter the set of business cycle variables to be solely described by growth on the industrial production index (IPI). In this scenario positive values of κ describe procyclical choices of leverage by the individual encouraging debt accumulation in expansionary episodes and deleveraging in recessions, and negative values of the parameter describe countercyclical choices of leverage yielding increases in debt accumulation in recessions and deleveraging in expansionary periods.

The second financial friction that we explore in this paper is the existence of a constant upper bound on the amount of financial leverage allowed to the individual. This restriction establishes a maximum debt ratio given by $\eta/(1+\eta)$ and entails a procyclical relationship between individual's debt and equity. In this scenario we assume that individuals prefer to borrow up to the maximum, determined by $\eta/(1+\eta)w_t$, and allocate these funds to consumption and investment in risky assets. This borrowing constraint is very similar to the financial frictions discussed in Kiyotaki and Moore (1997), and more recently, He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) as notable examples.

2.3 Optimal consumption rule

In order to study the optimal consumption rule we follow He and Krishnamurthy (2013), and assume that

$$c_t = \theta w_t \tag{8}$$

with $\theta > 0$ establishing the optimal ratio between individual's consumption and equity. These authors motivate the choice of this optimal consumption rule in a continuous time overlapping generations model. Expression (8), although restrictive from a macroeconomic perspective, can be motivated for long investment horizons under different theories, thus, early pioneering studies such as Friedman (1957) already notes this relation between consumption and wealth in his celebrated Permanent Income Hypothesis. Samuelson (1969) also observes such relationship between consumption and wealth in a Ramsey model of consumption. In Samuelson's seminal study the fraction of wealth consumed by the individual in each period differs depending on whether utility is of log form or reflects risk aversion. The case $\theta = 0$ corresponds to an individual only concerned with maximizing the multiperiod utility of wealth. Xiong (2001) and Gromb and Vayanos (2002) are examples of wealth models defined over long investment horizons proposed to assess the benefits of arbitrage trading activities.

Expressions (6), (8) and the budget constraint (5) establish the accumulation equation

$$w_{t+i} = \frac{1 + r^{eq}_{t+i}(\lambda, \kappa)}{1 + \theta} w_{t+i-1}, \tag{9}$$

with $r^{eq}_{t+i}(\lambda, \kappa) = r_{p,t+i}(\alpha_{t+i-1}) + \eta_{t+i-1}(r_{p,t+i}(\alpha_{t+i-1}) - r_{b,t+i})$ the return on equity. The notation of the return on equity simplifies to $r^{eq}_{t+i}(\lambda, \eta)$ if the leverage parameter is constant over time. Further, the accumulation equation entails the condition

$$w_{t+j} = \prod_{i=1}^{j} \left(\frac{1 + r^{eq}_{t+i}(\lambda, \kappa)}{1 + \theta} \right) w_t. \tag{10}$$

The next section shows that the structure of capital used by the individual for financing his investment portfolio has an effect on the optimal portfolio allocation of strategic investors exhibiting constant relative risk aversion.

2.4 Equilibrium conditions

The solution to the individual's asset allocation problem is obtained under two types of financial frictions. First, we consider individual's leverage to follow the functional form (7), and second, we consider a more restrictive condition imposing constant leverage on the individual's financial position.

DEFINITION. *Equilibrium is defined in this framework as a set of decisions* $\{\alpha_{ht}, \eta_t, c_t\}$ *parameterized by expressions* (4), (7) *and* (8), *respectively, such that*

(*i*) *Given the price processes, decisions solve the consumption-investment problem of an individual facing financial constraints.*

(*ii*) *Decisions satisfy the budget constraint* (5).

(*iii*) *The debt process satisfies* (6) *and the leverage parameter follows* (7) *or is constant.*

The solution to this partial equilibrium is determined by the parameters θ, λ and κ. Given our interest on the optimal asset allocation problem for long-term investors, we consider θ given and do not explore further the individual's optimal consumption decision. In this framework the individual's decision variables are the set of parameters λ and κ. The individual's maximization problem (1) becomes

$$\max_{\{\lambda,\kappa\}} \left\{ \sum_{j=1}^{K} \delta^j \frac{c_t^{1-\gamma}}{1-\gamma} E_t \left[\left(\prod_{i=1}^{j} (1 + r_{t+i}^{eq}(\lambda,\kappa)) \right)^{1-\gamma} \right] \right\}, \tag{11}$$

with $\delta = \beta(1+\theta)^{\gamma-1}$. The first order conditions of this optimization problem with respect to the vector of parameters λ_{hs}, with $h = 1, \dots, m$ and $s = 1, \dots, n$, provide a system of mn equations characterized by the following conditions:

$$E_t \left[\sum_{j=1}^{K} \delta^j \psi_{t,j}(z_s, \lambda_h) \right] = 0 \tag{12}$$

with

$$\psi_{t,j}(z_s, \lambda_h) = \left(\sum_{i=1}^{j} \frac{(1+\eta_{t+i-1}) z_{s,t+i-1} r_{h,t+i}^{e}}{1 + r_{t+i}^{eq}(\lambda,\kappa)} \right) \left(\prod_{i=1}^{j} (1 + r_{t+i}^{eq}(\lambda,\kappa)) \right)^{1-\gamma}, \tag{13}$$

where $r_{h,t+i}^{e} = r_{h,t+i} - r_{f,t+i}$ denotes the excess return on individual assets and η_{t+i-1} follows the specification (7).

The possibility of choosing the optimal amount of leverage in the financial position entails a further set of first order conditions obtained from maximizing the objective function with respect to κ. The additional set of optimality conditions is

$$E_t \left[\sum_{j=1}^{K} \delta^j \psi_{t,j}^{\kappa}(x, \kappa) \right] = 0 \tag{14}$$

with

$$\psi_{t,j}^{\kappa}(x,\kappa) = \left(\sum_{i=1}^{j} \frac{x_{s,t+i-1}(r_{t+i}^{p}(\lambda_h' z_{t+i-1}) - r_{b,t+i})}{(1+\exp(-\kappa' x_{t+i-1}))(1 + r_{t+i}^{eq}(\lambda,\kappa))} \right) \left(\prod_{i=1}^{j} (1 + r_{t+i}^{eq}(\lambda,\kappa)) \right)^{1-\gamma}. \tag{15}$$

The introduction of the vector of state variables z_t allows us to incorporate forecasts of the investment opportunity set in the optimal allocation of assets. These results can be refined by observing that the set of conditions (12) and (14) can be expressed in terms of an augmented set of unconditional expectations. To do this we need to assume that the conditioning information set determining the above conditional expectations is fully described by the vector $z_t = (z_{1,t}, z_{2,t}, \dots, z_{n,t})'$ of state variables with $z_{1,t} = 1$ and the rest of state variables being a set of $n-1$ macroeconomic and financial variables. Relevant examples of this estimation strategy applied to asset pricing and optimal portfolio theory can be found in Cochrane (1996) and Ait-Sahalia and Brandt (2001), among many others. Under these conditions, we have

$$E\left[\sum_{j=1}^{K}\beta^{j}\psi_{t,j}(z_s,\lambda_h)\otimes z_t\right]=0 \tag{16}$$

and

$$E\left[\sum_{j=1}^{K}\beta^{j}\psi_{t,j}^{\kappa}(x,\kappa)\otimes z_t\right]=0 \tag{17}$$

where $\otimes$ denotes element by element multiplication.

The equilibrium condition obtained from the maximization of the individual's multiperiod objective function simplifies if the leverage parameter is exogenously imposed on the individual. The financial constraint is given by the condition $b_{t+i} \leq f w_{t+i}$ with $f = \eta/(1+\eta)$. In this scenario the case of interest is when the restriction is binding. This situation corresponds to a dynamic borrowing constraint determined by the leverage parameter η and individual's equity. The optimization problem (11) is not maximized with respect to the leverage parameter that is assumed to be imposed by the lender on the borrower.

3 Econometric methods

This section discusses the empirical implementation of the optimality conditions derived in the preceding section. To do this we propose a simple econometric methodology for the estimation of the set of parameters λ and κ driving the optimal portfolio weights and leverage dynamics. The section also discusses inference procedures to test the correct specification of the parametric portfolio policy (4) and the parametric form of the function (7) driving the dynamics of the leverage parameter.

3.1 Estimation

Expressions (16) and (17) describe a system of $(mn+1)n$ Euler equations. These equations can be used to estimate the optimal portfolio weights characterized by the set of parameters λ and the optimal leverage position characterized by κ. GMM is a natural econometric technique to estimate $mn+1$ parameters using $(mn+1)n$ equations. A suitable empirical representation of the above system of Euler equations is

$$\widehat{\phi}_{h,s}(z_s,\lambda_h) = \frac{1}{T-K}\sum_{t=1}^{T-K}\left(\sum_{j=1}^{K}\beta^j\psi_{t,j}(z_s,\lambda_h)\otimes z_t\right) = 0 \tag{18}$$

and

$$\widehat{\phi}_{\kappa}(x,\kappa) = \frac{1}{T-K}\sum_{t=1}^{T-K}\left(\sum_{j=1}^{K}\beta^j\psi^{\kappa}_{t,j}(x,\kappa)\otimes z_t\right) = 0, \tag{19}$$

where T is the sample size used for estimating the model parameters. For each pair (h,s), condition (18) yields a $n\times 1$ vector of moment conditions denoted as

$$\widehat{\phi}^{(\widetilde{s})}_{h,s}(z_s,\lambda_h) = \frac{1}{T-K}\sum_{t=1}^{T-K}\left(\sum_{j=1}^{K}\beta^j\psi_{t,j}(z_s,\eta;\lambda_h)\ z_{\widetilde{s},t}\right) \tag{20}$$

with $\widetilde{s}=1,\ldots,n$ where $z_{1,t}=1$. Similarly, condition (19) yields a $n\times 1$ vector, $\widehat{\phi}^{(\widetilde{s})}_{\kappa}$, of moment conditions describing the empirical counterpart of the first order conditions (17) with respect to κ.

Let g_T be the $(mn+1)n\times 1$ vector that stacks each of the sample moments $\widehat{\phi}^{(\widetilde{s})}_{h,s}$ and $\widehat{\phi}^{(\widetilde{s})}_{\kappa}$ indexed by h, s and $\widetilde{s}$, with $h=1,\ldots,m$ and $s,\widetilde{s}=1,\ldots,n$. The idea behind GMM is to choose the vector $(\widehat{\lambda},\widehat{\kappa})$ so as to make the sample moments g_T as close to zero as possible. More formally, the GMM estimator of the vector (λ,κ) is defined as

$$(\widehat{\lambda},\widehat{\kappa}) = \underset{c_1\in\Lambda,c_2\in\kappa}{\arg\min}\ g_T(c_1,c_2)'\widehat{V}_T^{-1}g_T(c_1,c_2)$$

where $\widehat{V}_T$ is an $n(mn+1)\times n(mn+1)$, possibly random, non-negative definite weight matrix, whose rank is greater than or equal to $mn+1$. This matrix admits different representations. In a first stage $\widehat{V}_T$ can be the identity matrix or some other matrix, as for example, $I_{mn+1}\otimes Z'Z$, with I_{mn+1}

the identity matrix of dimension $mn+1$ and Z the $(T-K)\times n$ matrix corresponding to the state variables. In a second stage, to gain efficiency, this matrix is replaced by a consistent estimator of the asymptotic covariance matrix V_0 of the random vector $g_T(c_1, c_2)$.

The estimation procedure is simpler when the financial friction is established by a fixed leverage parameter. The set of optimality parameters is λ; the number of parameters to be estimated is mn, and the number of overidentified restrictions is mn^2.

3.2 Hypothesis tests

This section discusses different specification tests for statistically assessing the empirical suitability of the parametric specifications discussed above. To do this we exploit the overidentification of the system provided by the above set of first order conditions that allows us to apply standard specification $J-$tests proposed by Hansen (1982). This test assesses whether the estimates of the λ and κ parameters obtained from setting to zero $mn+1$ linear combinations of the $(mn+1)n$ equations conform with the rest of sample orthogonality conditions in $g_T(c_1, c_2)$. More specifically, the correct specification of the model implies that there are $(mn+1)(n-1)$ linearly independent combinations of $g_T(\widehat{\lambda}, \widehat{\kappa})$ that should be close to zero but are not exactly equal to zero. Let $s(\widehat{\lambda}, \widehat{\kappa}) = g_T(\widehat{\lambda}, \widehat{\kappa})' \widehat{V}_T^{-1} g_T(\widehat{\lambda}, \widehat{\kappa})$, that under the null hypothesis of correct specification of the model, satisfies

$$(T-K)s(\widehat{\lambda}, \widehat{\kappa}) \xrightarrow{d} \chi^2_{(mn+1)(n-1)}. \tag{21}$$

This test rejects the null hypothesis of correct specification of the models (4) and (7) if the test statistic is larger than the critical value obtained from a χ^2 distribution with $(mn+1)(n-1)$ degrees of freedom. Note that the convergence rate $T-K$ in (21) can be replaced by T given that the investment horizon K is assumed to be finite throughout.

The specification test (21) simplifies for the second type of financial frictions. In this case the test only assesses the specification (4) and the degrees of freedom of the χ^2 distribution are $mn(n-1)$.

4 Empirical application

In this section we focus on the optimal strategic asset allocation of a long term investor exhibiting constant relative risk aversion. We also assume no consumption ($\theta = 0$) by the individual. The individual's optimal portfolio policy rule follows the parametric function (4) with respect to a set of state variables describing the evolution of the economy. This modeling strategy is implemented for $\beta = 0.95$ and different parameterizations of the investor's objective function characterized by $K = 12$, 60 and 120 and $\gamma = 2$, 5 and 40. The borrowing interest rate is assumed to be the risk-free rate.

4.1 Descriptive analysis

Time variation of the investment opportunity set is described by a set of state variables that have been identified in the empirical literature as potential predictors of the excess stock and bond returns and the short-term ex-post real interest rates. The set of state variables are the detrended short-term interest rate (Campbell, 1991), the U.S. credit spread (Fama and French, 1989), the S&P 500 trend (At-Sahalia and Brandt, 2001) and the one-month average of the excess stock and bond returns (Campbell et al., 2003). The detrended short-term interest rate detrends the short-term rate by subtracting a 12-month backwards moving average. The U.S. credit spread is defined as the yield difference between Moody's Baa- and Aaa-rated corporate bonds. The S&P 500 momentum is the difference between the log of the current S&P 500 index level and the average index level over the previous twelve months. We demean and standardize all the state variables in the optimization process (Brandt et al, 2009).

Our data covers the period January 1980 to December 2010. We collect monthly data from Bloomberg on the S&P 500 and G0Q0 Bond Index. The G0Q0 Bond Index is a Bank of America and Merrill Lynch U.S. Treasury Index that tracks the performance of U.S. dollar denominated sovereign debt publicly issued by the U.S. government in its domestic market. We collect the Industrial Production Index (IPI) and Consumer Price Index (CPI) time series from the U.S. Federal Reserve and the yield of the Moody's Baa and Aaa-rated corporate bonds from the U.S. Federal Reserve. The nominal yield on the U.S. one-month risk-free rate reported in the Fama and French database is used as the risk-free rate and also as the borrowing rate on individuals' loans.

Table 1 reports the sample statistics of the annualized excess stock return, excess bond return and short-term ex-post real interest rates. The bond market outperforms the stock market during this period. In particular, the excess return on the bond index is higher than for the S&P 500 and exhibits a lower volatility entailing a Sharpe ratio almost three times higher for bonds than stocks. Additionally, the excess bond return has larger skewness and lower kurtosis. This anomalous outperformance of the G0Q0 index versus the S&P 500 is mainly explained by the last part of the sample and the consequences of the subprime crisis on the valuation of the different risky assets.

Table 2 shows the estimates of the seemingly unrelated regression estimation of the excess stock return, excess bond return and short-term ex-post real interest rate using as explanatory variables the detrended short-term interest rate, the U.S. credit spread, the S&P 500 trend and the one-month average of the excess stock and bond returns. These estimates allow us to obtain some insights on the dynamics of excess stock and bond returns and their variation over time linked to the state variables that we assume are driving the change in the investment opportunity set. A first conclusion that can be drawn from the estimated model, and in particular from the low R^2 statistics reported, is the difficulty in predicting excess asset returns.

4.2 Optimal asset allocation for strategic investors

The parameter estimates driving the optimal portfolio rules and dynamic leverage coefficients are estimated using a two-step Gauss-Newton type algorithm using numerical derivatives. In a first stage we initialize the covariance matrix $\widehat{V}_T$ with the matrix $I_{mn+1} \otimes Z'Z$, and in a second stage, after obtaining a first set of parameter estimates, we repeat the estimation replacing this matrix by a trimmed version of the empirical covariance matrix. In particular, we use a Newey-West estimator of the matrix V_0 with $K = 12$ lags.

[Insert Tables 1 to 3 about here]

Table 3 reports the estimates for the unconstrained case, $\eta = 0$, that constitutes the base scenario given by the absence of borrowing frictions, for $K = 12$, 60, 120 and $\gamma = 5$. The optimal strategic allocation to the S&P 500 index is found to be positively related to the one-month average of the excess stock and bond returns, and negatively related to the detrended short-term interest rate. The optimal strategic allocation to bonds is negatively related to all of the state variables. The test of overidentifying restrictions (21) shows that the strategic asset allocation model estimated under the investor's time horizon $K = 120$ is well specified (p-value larger than 0.20) but rejects the specifications $K = 12$ and 60. Therefore, the larger the investment period the better the specification of the model. This finding is also reflected on the magnitude of the p-values of the test.

4.2.1 Optimal asset allocation with dynamic leverage

In this section we wish to gauge the effect of the borrowing constraints on the coefficients that establish the optimal strategic asset allocation given an investor's risk profile. Our methodology also allows us to derive the optimal leverage position of the individual with respect to the business cycle. This is done by estimating the parameter κ under the assumption that individual's leverage is driven by the functional form (7).

The results in Table 4 shows that the long-term optimal allocation to the S&P 500 is positively related to the one-month average of the excess stock and bond returns across investment horizons. For $K = 120$, the long-term optimal allocation to the S&P 500 is also negatively related to the detrended short-term interest rate and positively related to the U.S. credit spread and the momentum variable. In contrast, the long-term optimal allocation to bonds is negatively related to all of the state variables. The test of overidentifying restrictions (21) reports lack of statistical evidence to reject the correct specification of the parametric choices (4) and (7) used to model long-term asset allocation. The comparison of the results in Tables 3 and 4 reveal significant differences in the values of the λ parameters associated to the S&P 500 and G0Q0 bond index. In particular, we observe a decrease in the magnitude of the parameter estimates for portfolios constructed using financial leverage.

The results in Table 4 also report a negative parameter estimate for κ across model specifications, that is more precisely estimated for longer investment horizons. These results provide, under our model assumptions, strong empirical evidence suggesting a negative relationship between the business cycle and the individual's leverage position. According to these estimates, an improvement in the economy described by growth on industrial production is reflected by lower levels of the leverage coefficient and the debt ratio. Similarly, an impoverishment in economic conditions, reflected in negative economic growth, implies higher leverage in the individual's financial position. Figure 1 plots the dynamics of the leverage parameter, η_t, and the debt ratio, $f(\eta_t)$, over time. Large values of these quantities match very well NBER recession periods.

[Insert Tables 4 to 5 about here]

We also analyze the effect of the dynamic leverage on the mean asset demands. Table 5 reports the mean optimal portfolio weights allocated to the S&P 500 and G0Q0 bond indices under different investment horizons ($K = 12$, 60 and 120) and a relative risk aversion coefficient $\gamma = 5$. This is done by comparing the base scenario characterized by absence of borrowing with the model reporting dynamic leverage positions. The differences between the optimal strategic portfolio weights corresponding to the unconstrained investor and the constrained investor with leverage dynamics described by (7) are illustrated in Figure 2 for $K = 120$ and $\gamma = 5$. The unconstrained investor's optimal strategic portfolio weights allocated to the S&P 500 and G0Q0 financial indices are very volatile and react very aggressively to changes of the investment opportunity set. Individuals facing highly levered financial positions cannot reach these positions and their optimal allocation assigns a lower fraction of wealth to the risky assets, limiting, therefore, the size of the short position on the one-month Treasury bill rate. Table 6 confirms this empirical observation by noting that the average optimal portfolio weight allocated to the S&P 500 and G0Q0 financial indices is, on average, reduced by 35% and 45%, respectively, under financial frictions. Interestingly, this reduction is more pronounced the shorter the individual's investment horizon.

[Insert Figures 1 to 2 about here]

Table 6 and 7 also illustrate the effect of financial frictions on the coefficients that establish the optimal strategic asset allocation for the long term investment horizon, $K = 120$, for different values of the relative risk aversion coefficient. Thus, Table 6 reports the parameter estimates of an investor with leverage parameter modeled by (7) and relative risk aversion coefficients $\gamma = 2$, 5 and 40. The test of overidentifying restrictions (21) reports lack of statistical evidence (p-values larger than 0.30) to reject the correct specification of (4) and (7) with the only exception of the model that considers $\gamma = 2$ that yields a p-value of zero. The λ parameter estimates show a consistent pattern across the different relative risk aversion coefficients: increasing values of λ are associated with decreasing values of γ. This empirical finding reveals the existence of a negative relationship between the degree of investor's risk aversion and the responsiveness of the individual's optimal portfolio to changes in the information set. This pattern is also observed for the optimal choice of leverage, reflected on the κ parameter, as γ decreases. More specifically, decreases in the coefficient of relative risk aversion are accompanied by stronger reactions of the individual's leverage position to business cycle dynamics.

[Insert Tables 6 to 7 about here]

Table 7 reports the mean optimal portfolio weights allocated to the S&P 500 and G0Q0 bond indices for $K = 120$ and $\gamma = 2$, 5 and 40. The purpose of this exercise is to compare the base scenario characterized by an investment portfolio fully funded with individual's equity with the scenario characterized by portfolios partially funded through debt. Table 7 shows that the mean optimal portfolio weights allocated to both financial indices diminish significantly for increasing values of the individual's leverage position.

4.2.2 Optimal asset allocation with constant leverage

In this section we study the optimal portfolio choice of an individual subject to a fixed leverage constraint. This scenario entails procyclical financial frictions characterized by debt processes that comove with individual's equity. We compute the optimal portfolio allocation for different values of the debt ratio establishing the borrowing constraint. For consistency with the preceding exercise, we consider $\beta = 0.95$, $K = 12$, 60 and 120, and $\gamma = 2$, 5 and 40.

Figure 3 plots the values of the λ parameters associated to the risky assets in the portfolio - $S\&P$ 500 index and G0Q0 bond index - with respect to different values of the borrowing constraint for $K = 120$ and $\gamma = 2$, 5 and 40. In order to assess the relationship between the borrowing constraint and the sensitivity of the investment portfolio to the state variables we consider different values of the debt ratio within the $(0, 1)$ interval. The strategic asset allocation is obtained in this case from the set of first order conditions (12) under the assumption that the parameter η is constant over time. The plot reveals that increases in investor's leverage and, hence, in the amount of debt raised by the individual, lower, in absolute value, the magnitude of the λ parameters associated to all of the state variables. Interestingly, the sign is preserved with respect to the base scenario. This result also provides evidence of a reduction in the responsiveness of the optimal portfolio weights to changes in the state variables, preventing individuals from engaging in very aggressive portfolio allocations even if the state variables could be anticipating an improvement of the investment opportunity set.

Figure 4 compares the strategic optimal portfolio weights of an unconstrained investor with those of a highly levered investor whose debt ratio equals $f(\eta) = 0.95$. The unconstrained optimal portfolio weights allocated to the S&P 500 and G0Q0 financial indices are very volatile and react very aggressively to changes of the investment opportunity set. This is also reflected on large short positions allocated to the one-month Treasury bill to fully benefit from existing profit opportunities on risky assets. In contrast, the optimal portfolio decisions of highly levered individuals are much more moderate and yield returns close to those obtained from fully investing on the risk-free asset even in the presence of potential profit opportunities obtained from investing on risky assets.

[Insert Figures 3 to 5 about here]

We also investigate the role of the investment horizon on the strategic asset allocation problem of an individual facing a fixed leverage position and a risk aversion coefficient $\gamma = 5$. Figure 5 plots the values of the λ parameters associated to the $S\&P$ 500 and G0Q0 bond indices for $K = 12$, 60 and 120. The results reported in this analysis are more heterogeneous than in the preceding exercises if the comparisons are done with respect to the investment horizon. Whereas there is a monotonically increasing relationship between the magnitude of the λ parameters and the investment horizon for almost all of the coefficients corresponding to the S&P 500 index, this is not the case for the coefficients gauging the dynamics of the G0Q0 bond index. In some cases, the sign of the parameters also varies with the investment horizon. Thus, variation in the U.S. credit spread has a positive impact on the allocation to the stock index for short investment horizons but a negative impact over longer horizons.

The trend S&P 500 state variable exhibits opposite findings; the sensitivity of the λ parameters to variation of this state variable switches from a negative to a positive sign as the investment horizon increases.

The overall analysis performed so far shows that increases in individual's leverage have the same qualitative effects as increases in the relative risk aversion coefficient. It is optimal for leveraged individuals to take more conservative investment positions even if their risk aversion coefficient is low. This finding is consistent with the results in Aiyagari and Gertler (1999). These authors show that the existence of the margin constraint increases the traders' effective degree of risk aversion, since they wish to avoid having to unload assets at discount prices. Furthermore, how effectively risk averse traders act depends on how close they are to violating their respective margin constraints.

4.3 Financial frictions and volatility of debt

The empirical section is completed with an study analyzing the volatility of the debt process over time. Debt volatility is interpreted as variation in the growth of debt. More specifically, if leverage is dynamic and driven by the specification (7), then growth of the debt process, $\Delta \ln b_t$, can be approximated by $\Delta \ln \eta_t + r_t^{eq}(\lambda, \kappa)$. In contrast, if financial leverage is constant over time, the process $\Delta \ln b_t$ can be approximated by $r_t^{eq}(\lambda, \eta)$.

This decomposition of $\Delta \ln b_t$ sheds important insights on the existence of effective ways of reducing the volatility of debt over time when individuals are allowed to set their optimal leverage positions. A simple strategy that provides support to the implementation of countercyclical macroprudential policies postulated in the recent macro-finance literature is to propose functional forms for the dynamic leverage coefficient that are negatively correlated with the individual's return on equity and such that the negative correlation can offset the increase in the variance due to entertaining a random leverage quantity. Our dynamic specification of the leverage parameter in terms of IPI growth yields, instead, a quantity $\Delta \ln \eta_t$ that is uncorrelated with the return on the individual's equity described in (9). This result entails, by construction, a higher variance of the dynamic case compared to the static leverage case and prevents individuals in our exercise from pursuing borrowing policies smoothing their debt over time at the same time as optimizing their leverage position. Figure 6 illustrates the dynamics of these processes for both scenarios.

[Insert Figure 6 about here]

The graph clearly reveals excess volatility of the dynamic case compared to the static cases. This can be quantified by computing the unconditional variance in each case. The empirical variance of $\Delta \ln b_t$ for the dynamic case is 0.367. The unconditional variance of the debt process for $f(\eta) = 0.5$ is 0.163 and for $f(\eta) = 0.95$ is 0.002.

5 Conclusions

An important contributor to the strong economic growth witnessed in the last forty years has been the development of sound credit markets providing access to funding to individuals. The easier access to credit has favoured consumption and investment. In this paper we have studied the influence that the existence of financial frictions limiting the access to credit may have in the construction of optimal portfolios for individuals with long-term investment horizons. To do this we have developed a novel methodology for solving the optimal asset allocation problem of strategic investors at the same time as deriving the dynamics of the individual's optimal financial leverage position.

The conclusions of this study are threefold. First, financial frictions limit the reaction of investors to changes in the investment opportunity set. Second, we observe that individuals that can optimize their financial leverage position choose countercyclical policies: increase their leverage during economic recessions and decrease it during economic expansions. Third, optimal portfolio weights and financial leverage are negatively related to the degree of investor's risk aversion and positively related to the investment horizon.

These conclusions are supported by the findings of our empirical application to a portfolio of stocks, bonds and cash. In particular, we uncover the existence of a monotonically decreasing relationship between the magnitude of the parameters driving the optimal portfolio weights and the amount of leverage allowed in the individual's financial position. This finding implies that the long-term optimal asset allocation of individuals with highly levered financial positions is less responsive to variations in market conditions and suggests that the existence of borrowing constraints increases the traders' effective degree of risk aversion by increasing the share of wealth invested on the risk-free asset. Furthermore, how effectively risk averse traders act also depends on the individual's relative risk aversion coefficient and the investment horizon characterizing his multiperiod objective function.

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TABLES AND FIGURES

Table 1: Sample statistics

	Mean	Volatility	Sharpe ratio	Skewness	Kurtosis
$R^e_{S\&P500}$	0.0266	0.131	0.21	-1.12	4.88
R^e_{Bonds}	0.0290	0.0566	0.51	0.15	2.17
r_f	0.0183	0.021		0.38	3.16

This table presents the sample statistics of the excess stock returns $\left(R^e_{S\&P500}\right)$, bond excess returns $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rates (r_f). The sample data covers the period January 1980 to December 2010. The return horizon is one month. The mean and the volatility are expressed in annualized terms.

Table 2: Seemingly unrelated regression estimation of the excess stock return, bond excess return and short-term ex-post real interest rates.

$R^e_{S\&P500}$	T_b	Def	Trend	ARP	R^2
$\beta_{R^e_{S\&P500}}$	-0.23	-0.13	-0.02	1.19	0.10
p-value	0.25	0.51	0.90	0.00	
R^e_{Bonds}	T_b	Def	Trend	ARP	R^2
β_{Bonds}	-0.02	-0.04	-0.01	-0.02	0.03
p-value	0.82	0.62	0.15	0.01	
r_f	T_b	Def	Trend	ARP	R^2
β_{r_f}	-0.05	0.25	0.19	-0.08	0.06
p-value	0.33	0.00	0.00	0.11	

This table presents the estimates of the seemingly unrelated regression estimation (SURE) of the excess stock return $\left(R^e_{S\&P500}\right)$, bond excess return $\left(R^e_{Bonds}\right)$ and short-term ex-post real interest rates $\left(r_f\right)$, using as explanatory variables the state variables: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 3: Optimal asset allocation. No financial constraints.

γ=5	K=12				K=60				K=120		
	$\alpha_{S\&P500}$		α_{Bonds}		$\alpha_{S\&P500}$		α_{Bonds}		$\alpha_{S\&P500}$		α_{Bonds}
λ_{Tb}	-0.23	λ_{Tb}	-1.06	λ_{Tb}	-0.30	λ_{Tb}	-1.28	λ_{Tb}	-1.20	λ_{Tb}	-0.92
t-stat	(-1.05)	*t-stat*	(-2.63)	*t-stat*	(-1.42)	*t-stat*	(-4.49)	*t-stat*	(-2.81)	*t-stat*	(-2.21)
λ_{Def}	0.21	λ_{Def}	-4.08	λ_{Def}	-0.18	λ_{Def}	-3.74	λ_{Def}	-0.35	λ_{Def}	-3.62
t-stat	(0.88)	*t-stat*	(-8.52)	*t-stat*	(-0.73)	*t-stat*	(-9.02)	*t-stat*	(-0.70)	*t-stat*	(-5.02)
λ_{Trend}	-0.18	λ_{Trend}	-3.10	λ_{Trend}	-0.14	λ_{Trend}	-3.59	λ_{Trend}	0.54	λ_{Trend}	-3.97
t-stat	(-0.75)	*t-stat*	(-5.62)	*t-stat*	(-0.67)	*t-stat*	(-8.30)	*t-stat*	(1.06)	*t-stat*	(-6.16)
λ_{ARP}	3.15	λ_{ARP}	-1.40	λ_{ARP}	3.95	λ_{ARP}	-1.09	λ_{ARP}	4.31	λ_{ARP}	-1.96
t-stat	(7.63)	*t-stat*	(-2.44)	*t-stat*	(13.73)	t-stat	(-2.93)	*t-stat*	(5.44)	t-stat	(-3.37)
$\chi 2$	68.96			$\chi 2$	56.94			$\chi 2$	46.85		
p-value	0.01			*p-value*	0.04			*p-value*	0.21		

This table reports the estimates of the optimal strategic asset allocation policy of an individual facing no financial constraints,$f(\eta) = 0$, and an investment horizon of K=120. The investment portfolio is comprised by the S&P 500 index, the G0Q0 bond index and the one-month U.S. Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function assuming γ=5, and a value of β=0.95. We consider the following state variables that drive the time-varying investment opportunity set: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 4: Optimal strategic asset allocation under time-varying financial constraints.

γ=5	K=12				K=60				K=120		
	$\alpha_{S\&P500}$		α_{Bonds}		$\alpha_{S\&P500}$		α_{Bonds}		$\alpha_{S\&P500}$		α_{Bonds}
λ_{Tb}	0.02	λ_{Tb}	-0.56	λ_{Tb}	-0.23	λ_{Tb}	-0.36	λ_{Tb}	-0.88	λ_{Tb}	-0.13
t-stat	(0.21)	*t-stat*	(-2.47)	*t-stat*	(-1.32)	*t-stat*	(-1.40)	*t-stat*	(-5.94)	*t-stat*	(-0.54)
λ_{Def}	0.15	λ_{Def}	-2.00	λ_{Def}	-0.03	λ_{Def}	-2.21	λ_{Def}	-0.29	λ_{Def}	-1.66
t-stat	(1.62)	*t-stat*	(-8.39)	*t-stat*	(-0.20)	*t-stat*	(-8.06)	*t-stat*	(-2.03)	*t-stat*	(-3.83)
λ_{Trend}	-0.11	λ_{Trend}	-1.55	λ_{Trend}	0.06	λ_{Trend}	-2.04	λ_{Trend}	0.82	λ_{Trend}	-2.31
t-stat	(-0.95)	*t-stat*	(-5.54)	*t-stat*	(0.38)	*t-stat*	(-5.08)	*t-stat*	(6.07)	*t-stat*	(-6.00)
λ_{ARP}	1.75	λ_{ARP}	-0.76	λ_{ARP}	2.30	λ_{ARP}	-0.43	λ_{ARP}	2.81	λ_{ARP}	-0.88
t-stat	(12.56)	*t-stat*	(-2.46)	*t-stat*	(9.56)	t-stat	(-1.58)	*t-stat*	(17.56)	t-stat	(-3.17)
κ	-0.08				-0.11				-0.27		
t-stat	(-1.94)				(-1.96)				(-3.74)		
χ^2	58.87			χ^2	59.15			χ^2	48		
p-value	0.07			*p-value*	0.06			*p-value*	0.31		

These tables report estimates of the optimal strategic asset allocation policy of an individual facing time-varying financial constraints as specified in equation (7) and investment horizons of *K*=12, 60, 120. The investment portfolio is comprised by the S&P500 index, the G0Q0 bond index and the one-month U.S. Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function assuming γ=5, given a value of β=0.95.We consider the following state variables that drive the time-varying investment opportunity set: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 5: Mean asset demands. No financial constraints vs. time-varying financial constraints. The effect of the investment horizon.

	No financial constraints			Time-varying financial constraints		
γ=5	$\alpha_{S\&P500}$	α_{Bonds}	α_{TB}	$\alpha_{S\&P500}$	α_{Bonds}	α_{TB}
K=12	1.36	3.07	-3.42	0.75	1.34	-1.09
K=60	1.50	2.33	-2.83	0.95	1.24	-1.19
K=120	0.82	3.40	-3.22	0.63	2.20	-1.83

This table reports the mean optimal allocation in percentage points to stocks, bonds and cash of an individual with an investment horizon of 12, 60 and 120 months. The case corresponding to individuals without financial constraints is reported on the left panel and the case corresponding to the existence of financial constraints is reported on the right panel. The optimal parametric portfolio policy rule is specified in equation (4) under the dynamic specification of the leverage parameter in equation (7). Individuals' preferences are characterized by a power utility function with CRRA coefficients γ=5 and a value of β=0.95. We consider the following state variables that drive the time-varying investment opportunity set: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 6: Optimal asset allocation under time-varying financial constraints. The effect of relative risk aversion.

γ=2				γ=5				γ=40			
	$\alpha_{S\&P500}$		α_{Bonds}		$\alpha_{S\&P500}$		α_{Bonds}		$\alpha_{S\&P500}$		α_{Bonds}
λ_{Tb}	-1.34	λ_{Tb}	0.14	λ_{Tb}	-0.88	λ_{Tb}	-0.13	λ_{Tb}	-0.10	λ_{Tb}	-0.06
t-stat	(-5.69)	*t-stat*	(0.30)	*t-stat*	(-5.94)	*t-stat*	(-0.54)	*t-stat*	(-3.80)	*t-stat*	(-1.28)
λ_{Def}	0.47	λ_{Def}	-4.75	λ_{Def}	-0.29	λ_{Def}	-1.66	λ_{Def}	-0.03	λ_{Def}	-0.22
t-stat	(2.01)	*t-stat*	(-9.10)	*t-stat*	(-2.03)	*t-stat*	(-3.83)	*t-stat*	(-1.66)	*t-stat*	(-3.75)
λ_{Trend}	0.66	λ_{Trend}	-4.50	λ_{Trend}	0.82	λ_{Trend}	-2.31	λ_{Trend}	0.10	λ_{Trend}	-0.36
t-stat	(2.83)	*t-stat*	(-8.09)	*t-stat*	(6.07)	*t-stat*	(-6.00)	*t-stat*	(1.76)	*t-stat*	(-5.05)
λ_{ARP}	5.78	λ_{ARP}	-0.36	λ_{ARP}	2.81	λ_{ARP}	-0.88	λ_{ARP}	0.36	λ_{ARP}	-0.11
t-stat	(22)	*t-stat*	(-0.79)	*t-stat*	(17.56)	t-stat	(-3.17)	*t-stat*	(8.57)	t-stat	(-1.36)
κ	-0.13				-0.27				-0.37		
t-stat	(-4.07)				(-3.74)				(-3.66)		
χ^2	86.77			χ^2	48			χ^2	43.80		
p-value	0.00			*p-value*	0.31			*p-value*	0.48		

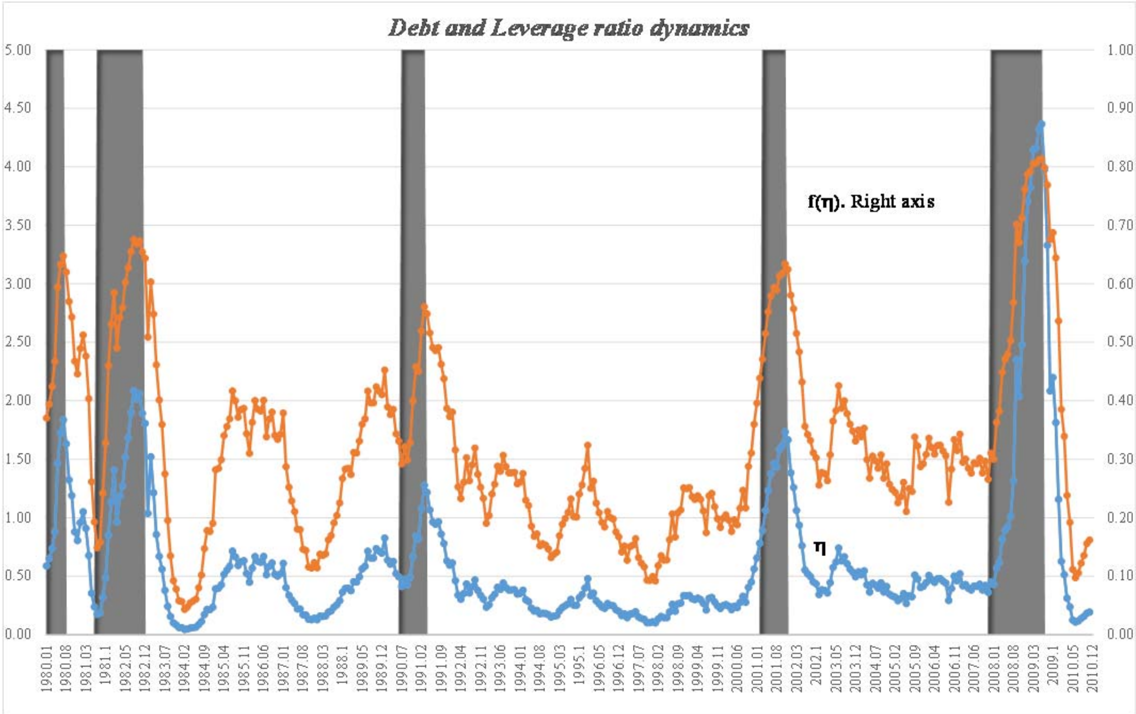
These tables report estimates of the optimal strategic asset allocation policy of an individual facing time-varying financial constraints as specified in equation (7) and investment horizons of *K*=120. The investment portfolio is comprised by the S&P500 index, the G0Q0 bond index and the one-month U.S. Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function given different values of γ=2, 5 and 40, and a value of β=0.95.We consider the following state variables that drive the time-varying investment opportunity set: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Table 7: Mean asset demands. No financial constraints vs. time-varying financial constraints. The effect of relative risk aversion.

	No financial constraints			Time-varying financial constraints		
K=120	$\alpha_{S\&P500}$	α_{Bonds}	α_{TB}	$\alpha_{S\&P500}$	α_{Bonds}	α_{TB}
γ=2	2.26	8.01	-9.27	1.73	3.34	-4.07
γ=5	0.82	3.40	-3.22	0.95	1.24	-1.19
γ=40	0.09	0.32	0.59	0.06	0.28	0.66

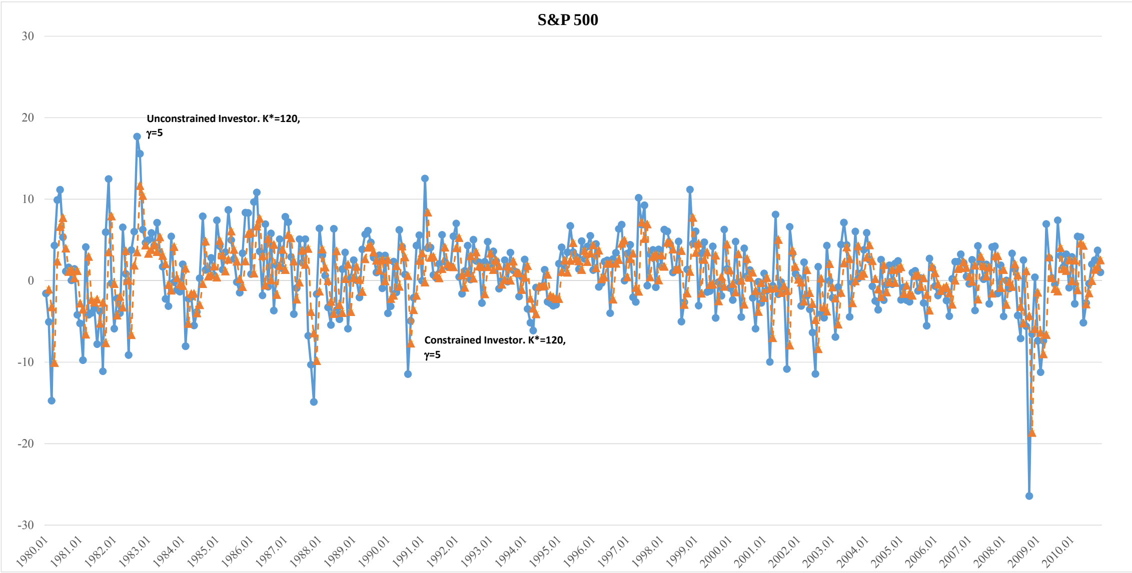
This table reports the mean optimal allocation in percentage points to stocks, bonds and cash of an individual with an investment horizon of 120 months. The case corresponding to individuals without financial constraints is reported on the left panel and the case corresponding to the existence of financial constraints is reported on the right panel. The optimal parametric portfolio policy rule is specified in equation (4) under the dynamic specification of the leverage parameter in equation (7). Individuals' preferences are characterized by a power utility function with CRRA coefficients γ=2,5 and 40 and a value of β=0.95. We consider the following state variables that drive the time-varying investment opportunity set: the detrended short-term interest rate (T_b), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

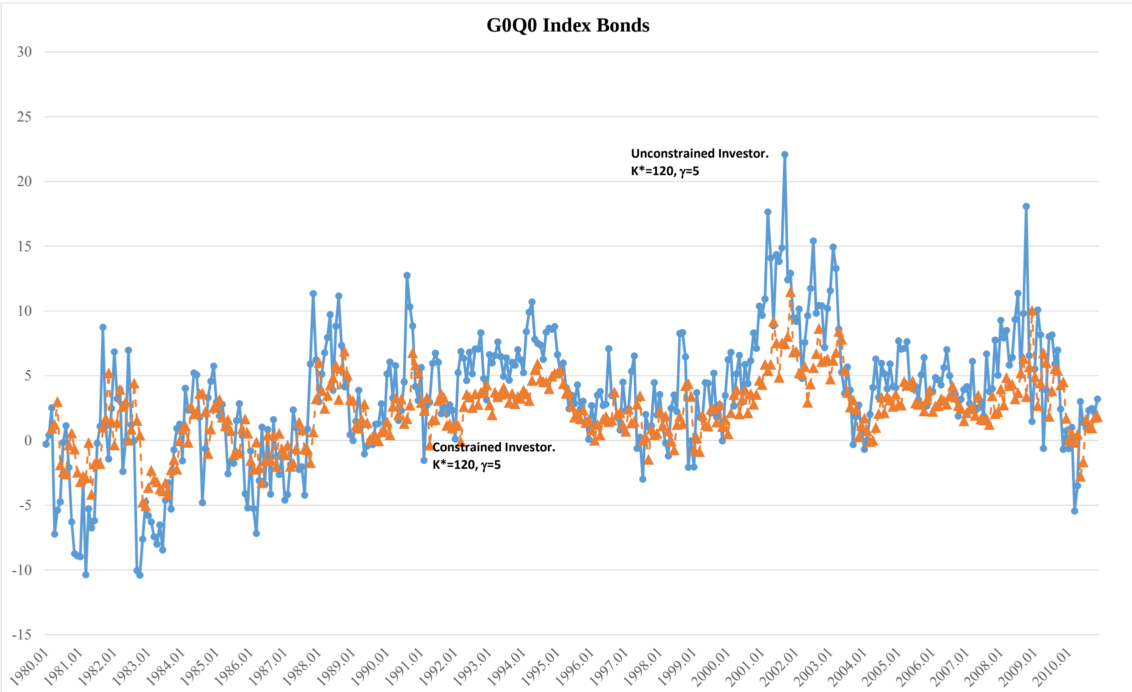
Figure 1: Borrowing constraint dynamics



The figure plots the estimated financial constraint dynamics as represented by η in equation (7) and the corresponding ratio of debt over wealth in each period, $f(\eta)$, satisfying $0 < f(\eta) < 1$. We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function using γ=5, a value of β=0.95 and an investment horizon K=120. The time-varying investment opportunity set is driven by the following state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Figure 2: Optimal Portfolio Weights under different time varying financial constraints.

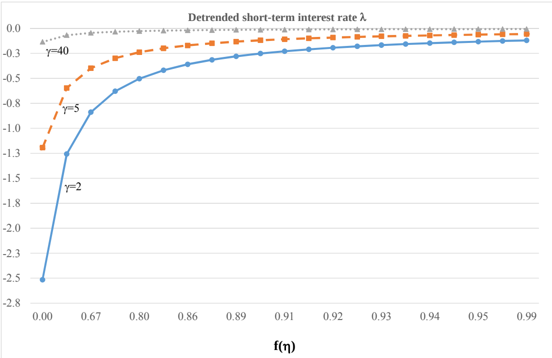


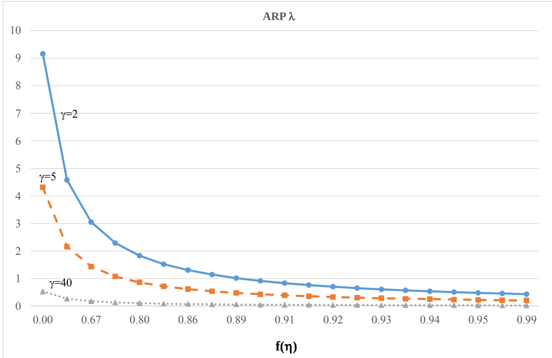


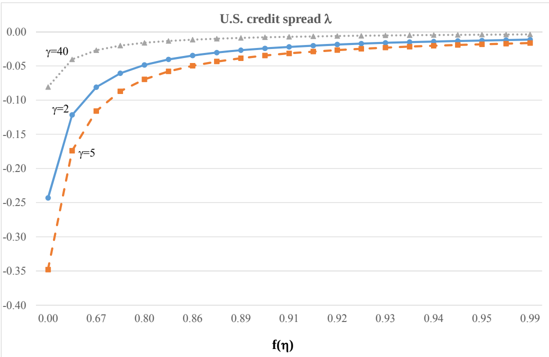
The figures plot the optimal portfolio weights as defined in equation (4) of the unconstrained investor and the constrained investor under time-varying borrowing constraints as defined in equation (7). We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function using γ=5, a value of β=0.95 and an investment horizon K= 120. The time-varying investment opportunity set is driven by the following state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

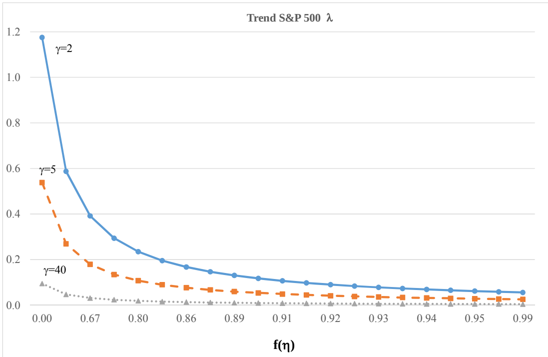
Figure 3: Parameter λ under constant financial constraints and different relative risk aversion coefficients.

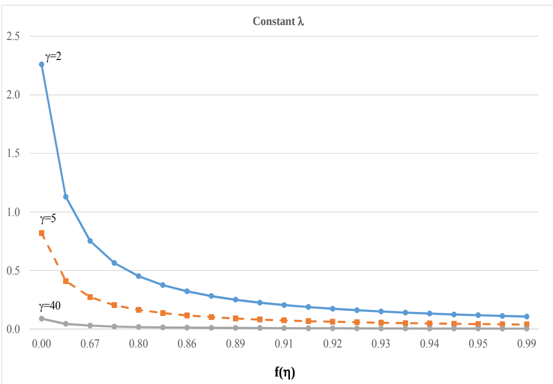
<u>S&P 500.</u>



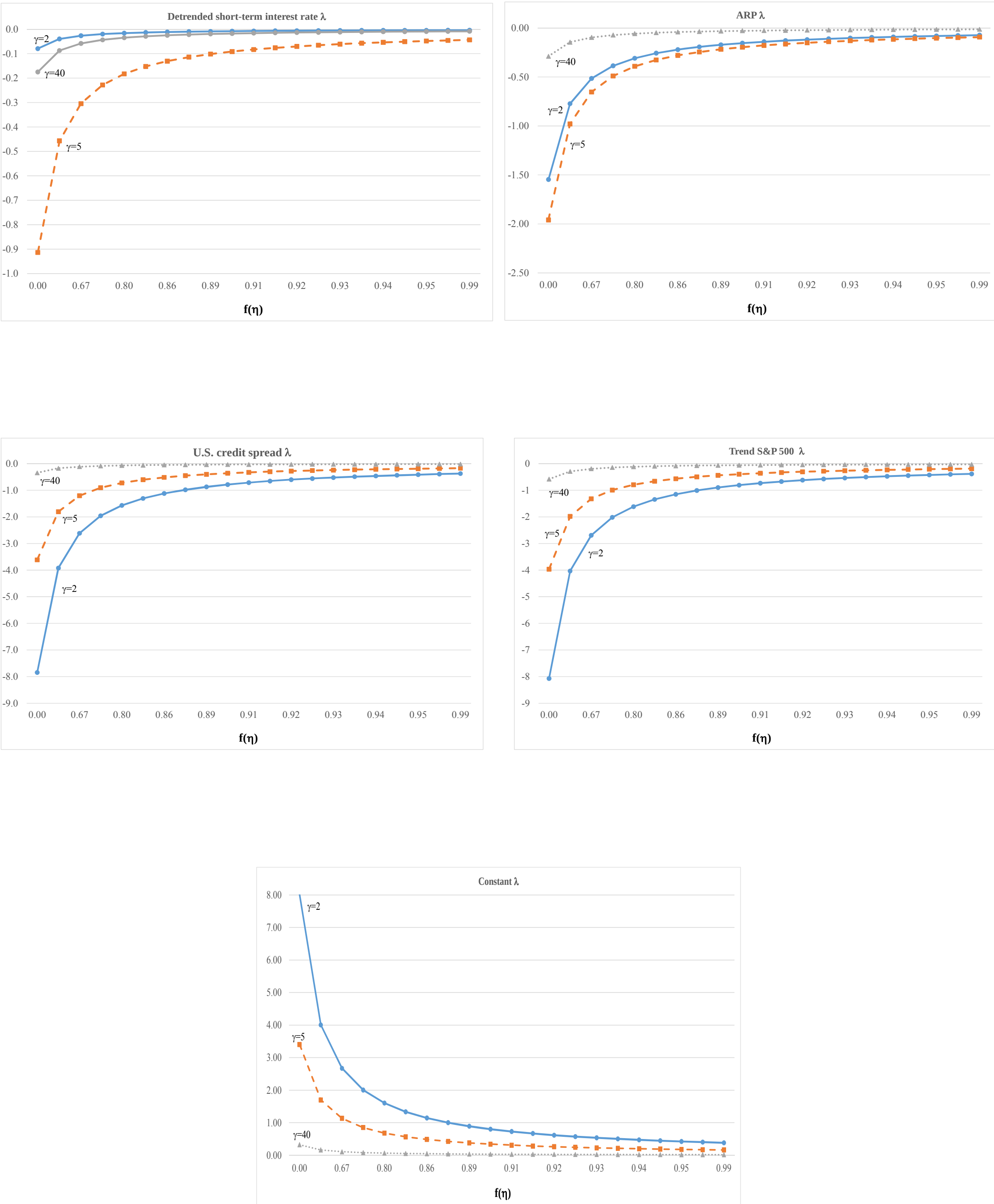






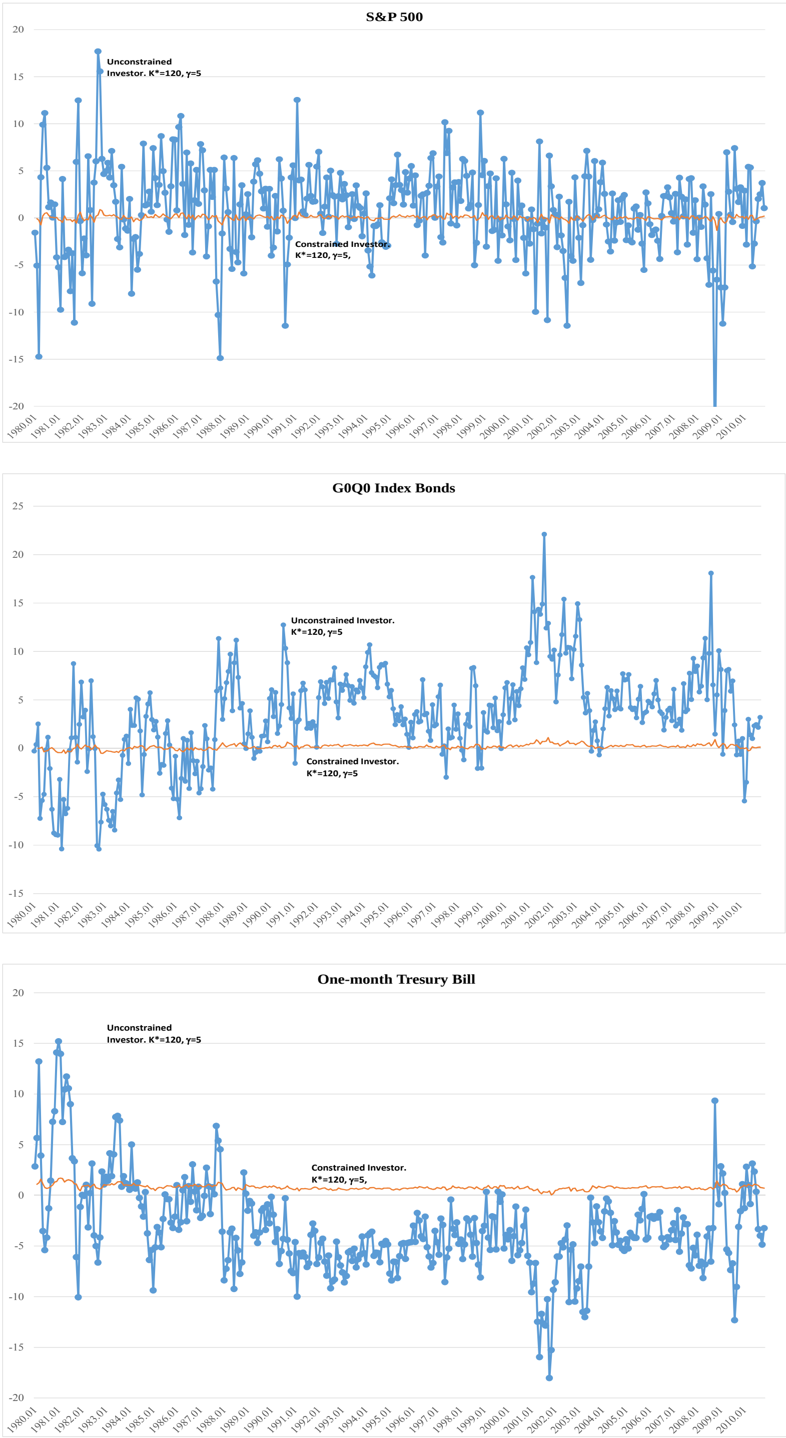


G0Q0 Index.



The different charts report the sensitivity of the vector of parameters λ in equation (4) to changes in the magnitude of the borrowing constraints faced by the investor. Financial constraints defined by the function $f(\eta)$ are constant and define the ratio of debt over wealth in each period $0 < f(\eta) < 1$.We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function using γ=5, 20 and 40, a value of β=0.95 and an investment horizon K=120. The time-varying investment opportunity set is driven by these state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

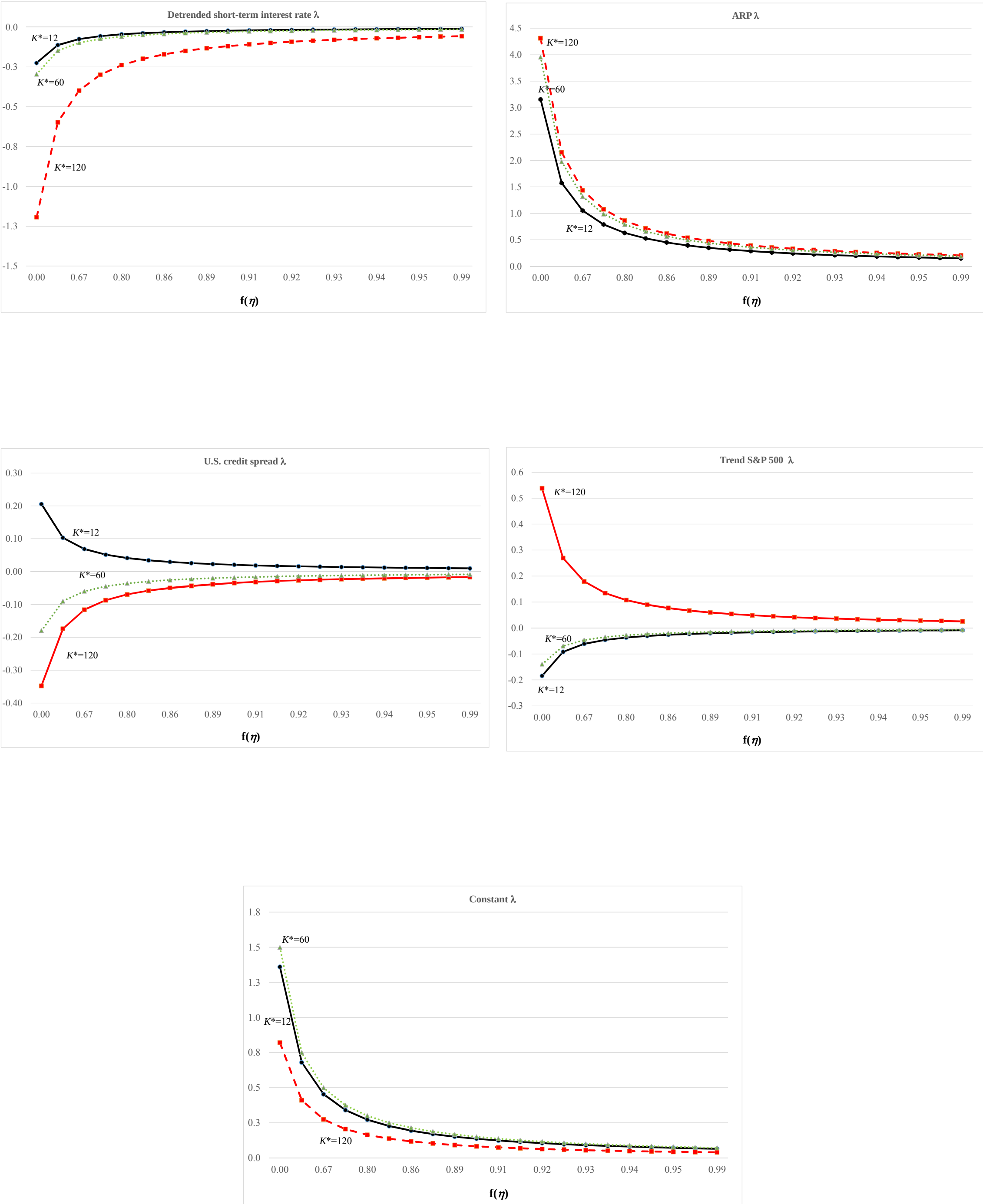
Figure 4: Optimal Portfolio Weights under different financial constraints for η constant.



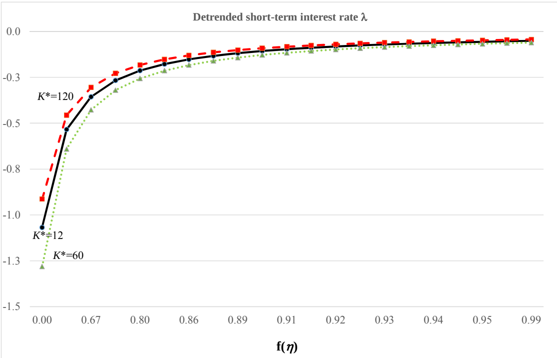
The figures plot the optimal portfolio weights as defined in equation (4) of the unconstrained investor and the constrained investor whose debt ratio in each period is constant, $f(\eta) = 0.95$. We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in the equation (4) and optimized for a CRRA utility function using γ=5, a value of β=0.95 and an investment time horizon K= 120. The time-varying investment opportunity set is driven by these state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

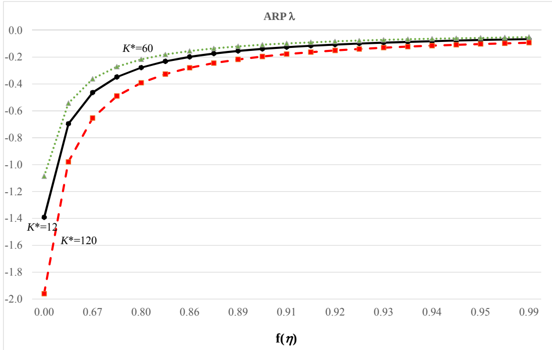
Figure 5: Parameter λ under constant financial constraints and different investment horizons.

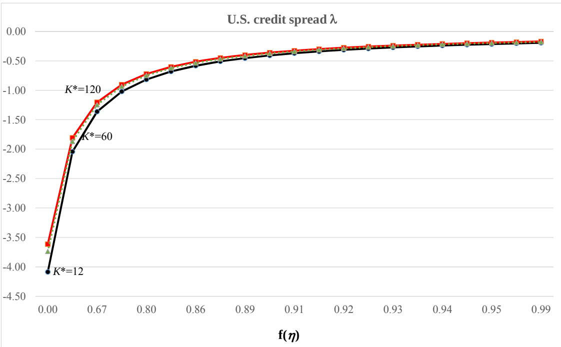
<u>**S&P 500.**</u>

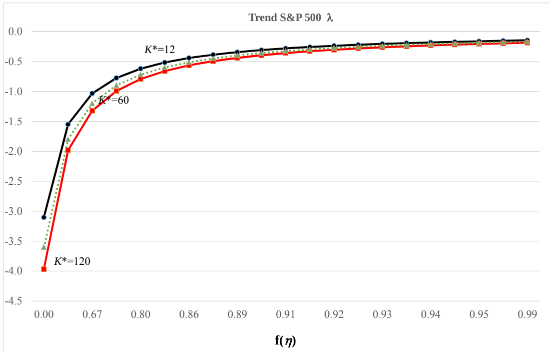


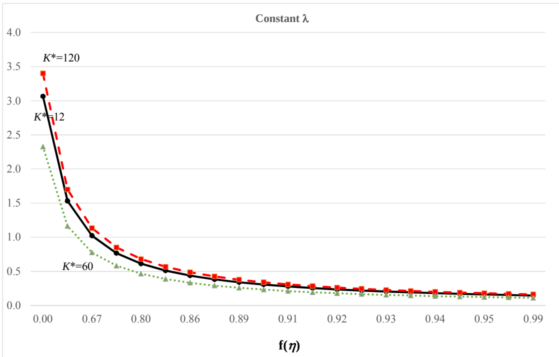
G0Q0 Index.





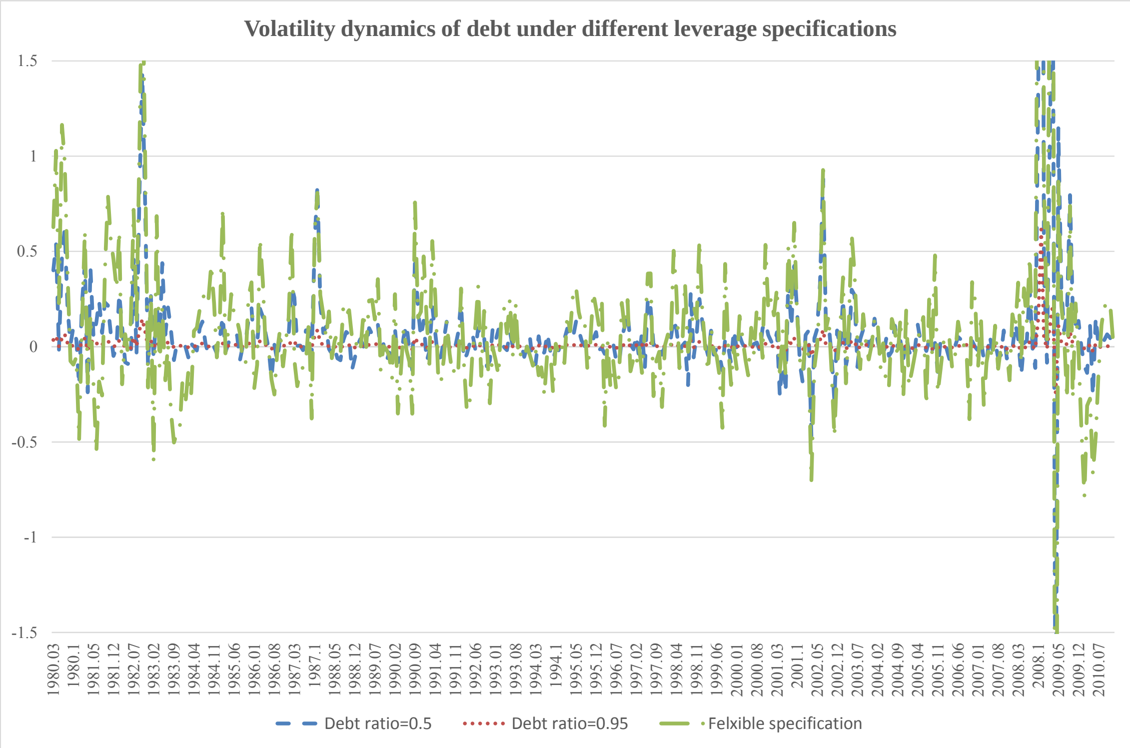






The different charts report the sensitivity of the vector of parameters λ in equation (4) to changes in the magnitude of the borrowing constraints faced by the investor. Financial constraints defined by the function $f(\eta)$ are constant and define the ratio of debt over wealth in each period $0 < f(\eta) < 1$.We consider an investor that can allocate her wealth among stocks, bonds and the one-month Treasury bill rate. The optimal portfolio rule is specified in equation (4) and optimized for a CRRA utility function using γ=5, a value of β=0.95 and investment horizons K=12, 60 and 120. The time-varying investment opportunity set is driven by these state variables: the detrended short-term interest rate (Tb), the U.S. credit spread (Def), the S&P 500 trend (Trend) and the one-month average of the excess stock and bond returns (ARP). We use monthly data from January 1980 to December 2010.

Figure 6. Volatility dynamics of debt under different leverage specifications.



This figure plots the debt growth rate under different leverage specifications: 1) constant debt ratio equals to 0.5, 2) constant debt ratio equals to 0.95 and 3) optimized debt ratio driven by the dynamic leverage coefficient (7). We use monthly data from January 1980 to December 2010.